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ON THE GEOMETRY OF SLANT AND PSEUDO-SLANT SUBMANIFOLDS IN A QUASI SASAKIAN MANIFOLDS

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Abstract. In this paper we introduce the notion of slant and pseudo-slant submanifolds of quasi-Sasakian manifold with the existence of slant submanifolds in quasi-Sasakian manifold.

Keywords: *slant submanifold, pseudo-slant submanifold, quasi-Sasakian manifold.*

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1. Introduction

The concept of slant submanifolds has been defined by B.Y. Chen [9] as natural generallization of both holomorphic and totally real immersions. After then many research articles have been appeared on the existence of these submanifolds in various know spaces. A. Lotta [12] investigated properties of slant submanifolds of an almost contact metric manifolds. L. Cabrerizo and others [6] investigated slant submanifolds of a Sasakian manifolds. Semi-slant submanifolds of Kaehler manifold N. Papaghich [13], as a naturel generalization of slant submanifolds. Bi-slant submanifolds was also introduced in a almost Hermitian manifold. Carriazo [7] defined and studied bi-slant submanifolds in an almost Hermitian manifold and gave the notion of pseudo-slant submanifold in an almost Hermitian manifold. V.A. Khan and others [11], defined and studied the contact version of pseudo-slant submanifold in a Sasakian manifold. CR-submanifold of a Kahlerian manifold is defined and studied by A. Bejancu [2]. On the other hand, semi-invariant submanifold of a Sasakian manifold was given and studied by A. Bejancu and N. Papaghiue [1]. These submanifolds are closely related to CR-submanifolds in a Kahlerian manifold. However the existence of the structure vector field implies some important changes.

2. Preliminaries

Let $\overline{M}$ be a real $(2n+1)$ dimensional differentiable manifold, endowed with an almost contact metric structure (f,ξ,η,g). Then we have

$$\text{(a) } f^2=-I+\eta\otimes\xi \qquad \text{(b) } \eta(\xi)=1 \qquad \text{(c) } \eta\circ f=0 \qquad \text{(d) } f(\xi)=0 \qquad (1)$$

(e) $\eta(X) = g(X, \xi)$ (f) $g(fX, fY) = g(X, Y) - \eta(X)\eta(Y)$

for any vector field X, Y tangent to $\overline{M}$, where I is the identity on the tangent bundle $T\overline{M}$, f is a tensor field of the type (1, 1), η is a 1-form, ξ is a vector field and g is a Riemannian metric on $\overline{M}$. Throughout the paper, all manifolds and maps are differentiable of class C^{∞}.We denote by $F(\overline{M})$ the algebra of the differentiable functions on $\overline{M}$ and by $F(E)$ the $F(\overline{M})$ module of the sections of a vector bundle E over $\overline{M}$.

The Niyembuis tensor field, denoted by N_f, with respect to the tensor field f, is given by

$$N_f(X,Y) = [fX, fY] + f^2[X,Y] - f[fX,Y] + f[X, fY], \quad \forall X, Y \in \Gamma(T\overline{M})$$

And the fundamental 2-form is given by

$$\Phi(X, Y) = g(X, fY) \tag{2}$$

The curvature tensor field of $\overline{M}$, denoted by $\overline{R}$ with respect to the Levi-Civita connection $\overline{\nabla}$, is defined by

$$\overline{R}(X,Y)Z = \overline{\nabla}_X \overline{\nabla}_Y Z - \overline{\nabla}_Y \overline{\nabla}_X Z - \overline{\nabla}_{[X,Y]} Z \qquad \forall X, Y \in \Gamma(T\overline{M}) \tag{3}$$

Definition 2.1 (a) An almost contact metric manifold $\overline{M}\ (f, \xi, \eta, g)$ is called normal if

$$N_f(X,Y) + 2d\eta(X,Y)\xi = 0 \qquad \forall X, Y \in \Gamma(T\overline{M}) \tag{4}$$

Or equivalently (cf. [12])

$$(\overline{\nabla}_{fX} f)Y = f(\overline{\nabla}_X f)Y - g(\overline{\nabla}_X \xi, Y)\xi \qquad \forall X, Y \in \Gamma(T\overline{M})$$

(b) The normal almost contact metric manifold $\overline{M}$ is called cosympletic if $d\Phi = d\eta = 0$.

Let $\overline{M}$ be an almost contact metric manifold $\overline{M}$. According to [8] we say that $\overline{M}$ is a quasi-Sasakian manifold if and only if ξ is a killing vector field and

$$(\overline{\nabla}_X f)Y = g(\overline{\nabla}_{fX} \xi, Y)\xi - \eta(Y)\overline{\nabla}_{fX} \xi \qquad \forall X, Y \in \Gamma(T\overline{M}) \tag{5}$$

Next we define a tensor field F of type (1, 1) by

$$FX = -\overline{\nabla}_X \xi \quad \forall X \in \Gamma(T\overline{M}) \tag{6}$$

Lemma 2.1 Let M be a quasi-Sasakian manifold. Then we have

(a) $(\overline{\nabla}_{\xi} f)X = 0 \ \forall X \in \Gamma(T\overline{M})$ (b) $foF = Fof$ (7)

(c) $F\xi = 0$ (d) $g(FX, Y) + g(X, FY) = 0 \quad \forall X, Y \in \Gamma(T\overline{M})$

(e) $\eta oF = 0$ (f) $(\overline{\nabla}_X F)Y = \overline{R}(\xi, X)Y \quad \forall X, Y \in \Gamma(T\overline{M})$

Let M be submanifold of a quasi-Sasakian manifold $\overline{M}$ and denote by N the unit vector field normal to M. Denote by the same symbol g the induced tensor metric on M, by ∇ the induced Levi-Civita connection on M and by $TM^{\perp}$ the normal vector bundle to M. The Gauss and Weingarten formula are

$$\overline{\nabla}_X Y = \nabla_X Y + h(X,Y) \tag{8}$$

$$\overline{\nabla}_X N = -A_N X + \nabla^{\perp}_X N, \ \forall X, Y \in \Gamma(TM), \tag{9}$$

where A_N is the shape operator with respect to the section N. It is known that

$$g(h(X,Y),\ N) = g(A_N X, Y) \qquad \forall X, Y \in \Gamma(TM) \ \ N \in \Gamma(TM^{\perp}) \tag{10}$$

For any $X \in TM$, $N \in T^{\perp}M$ we write

$$fX = PX + NX \qquad (PX \in TM,\ NX \in T^{\perp}M) \tag{11}$$

$$fN = tN + nN \qquad (tN \in TM,\ nN \in T^{\perp}M) \tag{12}$$

The submanifold M is invariant if N is identically zero. On the other hand, M is anti-invariant if T is identically zero. From (1) and (11), we have

$$g(X, TY) = -g(TX, Y) \tag{13}$$

for any $X, Y \in TM$. From now on, we put $Q = P^2$. We define

$$(\overline{\nabla}_X Q)Y = \nabla_X QY - Q\nabla_X Y \tag{14}$$

$$(\overline{\nabla}_X T)Y = \nabla_X TY - T\nabla_X Y \tag{15}$$

$$(\overline{\nabla}_X N)Y = \nabla^{\perp}_X NY - N\nabla_X Y \tag{16}$$

for any $X, Y \in TM$. In view of (8), (11), and (6) it follows that

$$\nabla_X \xi = -FX \tag{17}$$

$$h(X, \xi) = 0 \tag{18}$$

3. Pseudo-slant submanifolds of quasi-Sasakian manifolds

Definition 3.1. Let M be a submanifold of a quasi-Sasakian manifold $\overline{M}$. For each non-zero vector X tangent to M at x, the angle $\theta(x) \in [0, \frac{\pi}{2}]$, between ϕX and TX is called the slant angle or the Wirtinger angle of M. If the slant angle is constant for each $X \in \Gamma(TM)$ and $x \in M$, then the submanifold is also called the slant submanifold. If $\theta = 0$ the submanifold is invariant submanifold. If $\theta = \frac{\pi}{2}$ then it is called anti-invariant submanifold. If $\theta(x) \in [0, \frac{\pi}{2}]$, then it is called proper-slant submanifold .

Now, we will give the definition of pseudo-slant submanifold which are a generalization of the slant submanifolds.

Definition 3.2. Let $\overline{M}$ be a quasi-Sasakian manifolds and M an immersed submanifold in $\overline{M}$. We say that M is a pseudo-slant submanifold of quasi-Sasakian manifolds $\overline{M}$ if there exist two orthogonal distributions D and $D^{\perp}$ on M such that

(a) TM admits the orthogonal direct decomposition $TM = D \oplus D^{\perp}$, $\xi \in \Gamma(D)$

(b) The distribution $D^{\perp}$ is anti-invariant i.e., $\phi(D^{\perp}) \subseteq T^{\perp}M$,

(c) The distribution D is a slant with slant angle $\theta \neq 0$, that is, the angle between $\phi(D)$ and D is a constant.

From the definition, it is clear that if $\theta = 0$, then the pseudo-slant submanifold is a semi invariant submanifold. On the other hand, if $\theta = \frac{\pi}{2}$, submanifold becomes an anti- invariant.

We suppose that M is a pseudo-slant submanifold of quasi-Sasakian manifolds $\overline{M}$ and we denote the dimensions of distributions D and $D^{\perp}$ by d_1 and d_2, respectively, then we have the following cases:

(a) If $d_2 = 0$ then M is an anti-invariant submanifold,

(b) If $d_1 = 0$ and $\theta = 0$, then M is an invariant submanifold,

(c) If $d_1 = 0$ and $\theta \neq 0$, then M is a proper slant submanifold with slant angle θ,

(d) If $d_1.d_2 \neq 0$ and $\theta \in [0, \frac{\pi}{2}]$ then M is a proper pseudo-slant submanifold.

Theorem 3.1. Let M be a submanifold of a quasi-Sasakian manifold $\overline{M}$ such that $\xi \in TM$. Then M is slant if and only if there exists a constant $\lambda \in [0, 1]$ such that

$$P^2 = -\lambda\{I - \eta \otimes \xi\} \tag{19}$$

Furthermore, in such a case if θ is the slant angle of M then $\lambda = \cos^2\theta$.

Proof: Let M be a slant submanifold of a quasi-Sasakian manifold $\overline{M}$ with slant angle $\theta.\xi \in TM$. Then

$$\cos\theta = \frac{g(PX,\ fX)}{\|PX\|\|fX\|} = \frac{\|PX\|}{\|fX\|} = cons\tan t \tag{20}$$

for $X \in \Gamma(TM)$. In this case, from (20) we have

$$\cos\theta\|fX\| = \|PX\| \tag{21}$$

If we take the square (21) we get

$$g(PX,\ PX) = g(fX,\ fX)\cos^2\theta \tag{22}$$

Moreover,

$$\cos\theta = \frac{g(PX,\ fX)}{\|PX\|\|fX\|} = \frac{g(fX,\ PX)}{\|PX\|\|fX\|} = -\frac{g(X,\ fPX)}{\|PX\|\|fX\|} \tag{23}$$

$$= -\frac{g(X,\ P^2X)}{\|PX\|\|fX\|}$$

all vector field X. Then from (1), (20) and (23) we find

$$\cos^2\theta\{g(X - \eta(X)\xi, X\} = -g(P^2X,\ X)$$

In this case, we have

$$P^2X = -\cos^2\theta\{X - \eta(X)\xi\}$$

for any vector field $X \in TM$ That is, for all $X \in TM$

$$P^2 = -\lambda\{I - \eta \otimes \xi\}, \qquad \lambda = \cos^2\theta \tag{24}$$

Conversely, we now assume that the (19) holds. Then from (7) and (19) we obtain

$$\cos\theta = \frac{g(PX,\ \phi X)}{\|PX\|\|\phi X\|} = \frac{g(P^2X,\ X)}{\|PX\|\|\phi X\|} = -\frac{\lambda g(X - \eta(X)\xi,\ X)}{\|PX\|\|\phi X\|} = -\lambda\frac{\|\phi X\|}{\|PX\|}$$

Also by using (11), we conclude that

$$\lambda = \cos^2\theta \tag{25}$$

where θ is constant because λ is a constant, and so M is slant.

Corollary 3.1. Let M be a slant submanifold of a quasi-Sasakian manifold $\overline{M}$ with slant angle θ. Then for any $X, Y \in \Gamma(TM)$ we have

$$g(PX,\ PY) = \cos^2\theta\left(g(X,Y) - \eta(X)\eta(Y)\right) \tag{26}$$

$$g(NX,\ NY) = \sin^2\theta\left(g(X,Y) - \eta(X)\eta(Y)\right) \tag{27}$$

Proof: We can find with direct calculations using Theorem 3.1.

Proposition 3.1. Let M be a slant submanifold of a quasi-Sasakian manifold $\overline{M}$. Then $\nabla Q = 0$ if and only if M is an anti-invariant submanifold of $\overline{M}$.

Proof: We denote the slant angle of M by θ. For any $X, Y \in \Gamma(TM)$, since $Q = P^2$ and M is a slant submanifold, we have

$$Q(\nabla_X Y) = -\cos^2\theta\{\nabla_X Y - \eta(\nabla_X Y)\xi\} \tag{28}$$

On the other hand, differentiating covariant derivative of $QY = -\cos^2\theta\{Y - \eta(Y)\xi\}$ in the direction of X and using (3) and (6), we obtain

$$\nabla_X QY = -\cos^2\theta\{\nabla_X Y - X\eta(Y)\xi - \eta(Y)\nabla_X \xi\}$$

$$= -\cos^2\theta\{\nabla_X Y - \eta(\nabla_X Y)\xi + g(Y, FX) + \eta(Y)FX\} \tag{29}$$

On the other hand, from (14), (28) and (29) we have

$$(\nabla_X Q)Y = \nabla_X QY - Q\nabla_X Y = -\cos^2\theta\{g(Y, FX) + \eta(Y)FX\} \tag{30}$$

which implies that $\nabla Q = 0$ if and only if $\theta = \frac{\pi}{2}$. Thus M is an anti-invariant submanifold.

Lemma 3.1. Let M be a pseudo-slant submanifold of a quasi-Sasakian manifold $\overline{M}$. Then at each point p of M, Q_p has only one eigenvalue $\lambda = \cos^2\theta$.

Proof. The proof is similar to that in [3], so we omit it.

Theorem 3.2. Let M be a submanifold of a quasi-Sasakian manifold $\overline{M}$ such that $\xi \in TM$. Then M is a slant submanifold if and only if

(a) The endomorphism $Q|_D$ has only one eigenvalue at each point of M.

(b) There exists a function $\lambda : M \to (0, 1]$ such that

$$(\nabla_X Q)Y = -\lambda\{g(Y, FX) + \eta(Y)FX\} \tag{31}$$

for any $X, Y \in \Gamma(TM)$. Furthermore, if θ is the slant angle of M, then it satisfies $\lambda = \cos^2\theta$.

Proof. If M is a slant submanifold of a quasi-Sasakian manifold $\overline{M}$ with slant angle θ, then Lemma 3.1 and (30) imply that the relations (a) and (b) are satisfied.

Conversely, let $\lambda(p)$ be the only eigenvalue of $Q|_D$ at each point $p \in M$. Moreover, let $Y \in \Gamma(D)$ be a unit vector associated with λ, that is, $QY = \lambda Y$. Then by virtue of (b) and differentiating the covariant derivative of $QY = \lambda Y$ in the direction of X. We have

$$\nabla_X(QY) = \nabla_X(\lambda)Y + \lambda\nabla_X Y$$

$$\nabla_X(Q)Y + Q(\nabla_X Y) = X(\lambda)Y + \lambda\nabla_X Y$$
$$-\cos^2\theta\{g(Y,FX) + \eta(Y)FX\} + Q(\nabla_X Y) = X(\lambda)Y + \lambda\nabla_X Y$$
$$X(\lambda)g(Y,Y) = -g(\lambda\nabla_X Y, Y) + g(Q\nabla_X Y, Y)$$
$$= g(\nabla_X Y, \lambda Y) - g(\nabla_X Y, QY) = 0$$

that is, λ is a constant function. In order to prove that M is a slant submanifold, it is enough to show that there is a constant μ such that $Q = -\mu\{I - \eta\otimes\xi\}$. For $X \in \Gamma(TM)$ we can write $X = -\overline{X} + \eta(X)\xi$, where $\overline{X} = -X + \eta(X)\xi \in \Gamma(D)$. So we have $QX = Q\overline{X}$ and $QX = \lambda X$ because $Q|_D = \lambda I$, that is, $QX = \lambda(-X + \eta(X)\xi)$. Taking $\lambda = \mu$, we get the desired assertion.

Theorem 3.3. Let M be a pseudo-slant submanifold of a quasi-Sasakian manifold $\overline{M}$. Then

$$A_{fY}X = A_{fX}Y \tag{32}$$

for all $X, Y \in D$.

Proof: In view of (10),

$$g(A_{fY}X, Z) = g(h(X,Z),\ fY) = -g(fh(X,\ Z),\ Y) \tag{33}$$

By virtue of (8), (33) reduces to

$$g(A_{fY}X, Z) = -g(f\overline{\nabla}_Z X,\ Y) + g(f\nabla_Z X,\ Y)$$
$$= -g(f\overline{\nabla}_Z X,\ Y) \quad \text{since } f\nabla_Z X \in T^{\perp}M$$
$$= g((\overline{\nabla}_Z f)X,\ Y) - g(\overline{\nabla}_Z fX,\ Y) \tag{34}$$

Now, for $X \in D,\ fX \in T^{\perp}M$ Hence, from (9) we have

$$(\overline{\nabla}_Z f)X = -A_{fX}Z + \nabla^{\perp}_Z fX \tag{35}$$

Combining (34) and (35), we obtain

$$g(A_{fY}X, Z) = g((\overline{\nabla}_Z f)X,\ Y) + g(A_{fX}Z,\ Y) \tag{36}$$

Since $h(X,Y) = h(Y,X)$, if follows from (10) that

$$g(A_{fX}Z, Y) = g(A_{fX}Y, Z)$$

Hence, from (36) we obtain, with the help of (5),

$$g(A_{fY}X, Z) - g(A_{fX}Z, Y) = g(g(\overline{\nabla}_{fZ}\xi,\ X)\xi - \eta(X)\overline{\nabla}_{fZ}\xi,\ Y)$$
$$= \eta(Y)g(\overline{\nabla}_{fZ}\xi,\ X)\xi - \eta(X)g(\overline{\nabla}_{fZ}\xi,\ Y) \tag{37}$$

Since $X, Y, Z \in D$ an orthonormal distribution to the distribution $\langle\xi\rangle$ it follows that $\eta(X) = \eta(Y) = 0$ Therefore, the above equation reduces to

Theorem 3.4. Let M be a pseudo-slant submanifold of a quasi-Sasakian manifold $\overline{M}$. Then the distribution $D \oplus \langle\xi\rangle$ is integrable.

Proof: Since $h(X,\ Y) = h(Y,\ X)$, in view of (8) we see that

$$\nabla_X Y - \nabla_Y X = \overline{\nabla}_X Y - \overline{\nabla}_Y X \tag{38}$$

Let $X, \in D, Y, \in D^{\perp}$, then

$$(\overline{\nabla}_X g)(Y,Z) = \overline{\nabla}_X g(Y,Z) - g(\overline{\nabla}_X Y,\ Z) - g(Y,\ \overline{\nabla}_X Z)$$

or

$$0 = 0 - g(\overline{\nabla}_X Y,\ Z) - g(Y,\ \overline{\nabla}_X Z).$$

Hence

$$g(\overline{\nabla}_X Y,\ Z) = -g(Y,\ \overline{\nabla}_X Z). \tag{39}$$

Now

$$\begin{aligned} g([X,\xi],\ TZ) &= g(\nabla_X \xi - \nabla_\xi X,\ TZ) \\ &= g(\overline{\nabla}_X \xi - \overline{\nabla}_\xi X,\ TZ) \\ &= g(\overline{\nabla}_X \xi,\ TZ) - g(\overline{\nabla}_\xi X,\ TZ) \end{aligned} \tag{40}$$

Since $X \in D,\ Y \in D^{\perp}$ where D and $D^{\perp}$ are two orthogonal distributions and D is anti-invariant, in view of (17), (39) we obtain from (40)

$$g([X,\xi],\ TZ) = g(\overline{\nabla}_\xi TZ,\ X) \tag{41}$$

In view of (5),

$$(\overline{\nabla}_\xi f)Y = 0 \tag{42}$$

In virtue of (11) and (42), equation (41) yields

$$g([X,\xi],\ TZ) = 0$$

Hence $[X,\ \xi] \in D,\ X, \in D:$ Therefore, the distribution $D \oplus \langle \xi \rangle$ is integrable.

Theorem 3.5. Let M be a pseudo-slant submanifold of a quasi-Sasakian manifold $\overline{M}$. Then for any $X, Y \in D \oplus D^{\perp}$.

$$g([X,Y],\ \xi) = 0 \tag{43}$$

Proof:

$$g([X,Y],\ \xi) = g(\nabla_X Y,\ \xi) - g(\nabla_Y X,\ \xi) \tag{44}$$

In view of (39) we have from above

$$g([X,Y],\ \xi) = -g(\nabla_X \xi,\ Y) + g(\nabla_Y \xi,\ X) \tag{45}$$

By (17), (45) yields

$$g([X,Y],\ \xi) = 0$$

Theorem 3.6. Let M be a pseudo-slant submanifold of a quasi-Sasakian manifold $\overline{M}$. Then the anti-invariant distribution D is integrable.

Proof: For any $X, \in TM$, let

$$X = P_1 X + P_2 X + \eta(X)\xi \tag{46}$$

where $P_i \quad i = 1, 2$ are projection maps on the distribution D_i From (46) it follows that

$$fX = NP_1 X + TP_2 X + NP_2 X$$

$$TX = TP_2 X\ , \quad NX = NP_1 X + NP_2 X$$

Now for any $X, Y \in D$ and $Z \in D^{\perp}$

$$g([X,Y],\ TZ) = g([X,Y],\ TP_2 Z) = -g(f[X,\ Y],\ P_2 Z) \tag{47}$$

Now

$$\begin{aligned} f[X,\ Y] &= f\nabla_X Y - f\nabla_Y X \\ &= f\overline{\nabla}_X Y - f\overline{\nabla}_Y X \\ &= \overline{\nabla}_X fY - (\overline{\nabla}_X f)Y - \overline{\nabla}_Y fX + (\overline{\nabla}_Y f)X \end{aligned} \tag{48}$$

In view of (5) and (9) and keeping in mind that $g(U,V)=0$ for $U \in D$ and $V \in D^{\perp}$, we obtain from

$$g([X,Y],TP_2Z) = -g(A_{fX}Y - A_{fY}X + \eta(Y)\overline{\nabla}_{fX}\xi + \eta(X)\overline{\nabla}_{fY}\xi, P_2Z) \qquad (49)$$

For $X,Y \in D$ we get $\eta(X)=\eta(Y)=0$. Hence Theorem 3.3 and the above equation yield $g([X,Y],TZ)=0$, that is, $[X,Y]\in D$ for $X,Y \in D$. Therefore the distribution D is integrable.

4. Conclusion

Here our objective is to introduce pseudo-slant submanifolds of quasi Sasakian manifolds and study the notion of slant pseudo-slant submanifolds of quasi Sasakian manifolds. Integrability conditions of the distributions on these submanifolds are worked out. Some interesting results regarding such manifolds have also been deduced. The results obtained in this paper can be used in many problems of dynamical system and critical point theory.

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